Online Appendix

Re-estimating Potential GDP:

New Evidence on Output Hysteresis

Diego Anzoategui
Rutgers

Min Kim
Rutgers

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1 Baseline Model Details

1.1 Household

The representative household has the following utility function.

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t Z_t \left[\log\left(C_t - h\bar{C}_{t-1}\right) - \chi \frac{N_t^{1+\varphi}}{1+\varphi} \right]$$

where C_t and N_t represent consumption and labor, and $\bar{C}_{t-1}$ is the aggregate consumption at time $t-1$. $Z_t \equiv \exp(\sigma_z z_t)$ is a preference shock where z_t follows an exogenous process. The household is in charge of making consumption/saving decisions. The budget constraint it faces is the following

$$P_t C_t + B_{t+1} = W_t(i) N_t + R_{t-1} B_t + T_t$$

where T_t is profits from firms, B_t is bond holdings, R_{t-1} is the nominal interest rate, and W_t denotes the nominal wage. The Euler equation of the representative household is given by,

$$\begin{aligned} 1 &= \mathbb{E}_t \left\{ \beta \frac{Z_{t+1}}{Z_t} \frac{MU_{t+1}}{MU_t} \frac{R_t}{\Pi_{t+1}} \right\} \\ MU_t &= \frac{1}{C_t - h\bar{C}_{t-1}} \end{aligned}$$

where Π_t is the gross inflation rate and MU_t is the marginal utility from consumption.

1.2 Firms

Final good producers combine intermediate goods to produce the final goods with technology,

$$Y_t = \left(\int_0^1 Y_{kt}^{\frac{\varepsilon-1}{\varepsilon}} dk \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

Cost minimization yields the following demand for intermediate goods,

$$Y_{kt} = \left(\frac{P_{kt}}{P_t} \right)^{-\varepsilon} Y_t$$

where

$$P_t = \left(\int_0^1 P_{kt}^{1-\varepsilon} dk \right)^{\frac{1}{1-\varepsilon}}$$

Intermediate good producers produce a continuum of varieties denoted by k. The technology is,

$$Y_{kt} = A_t \left(\int_0^1 N_{kt}(i)^{\frac{\varepsilon_w - 1}{\varepsilon_w}} di \right)^{\frac{\varepsilon_w (1-\alpha)}{\varepsilon_w - 1}}$$

where A_t is an exogenous TFP shock that follows the process,

$$\log A_t = g_y + \log A_{t-1} + \sigma_a \varepsilon_{at}$$

The labor cost minimization problem is,

$$\min \int_0^1 W_t(i) N_{kt}(i) di$$

$$s.t.$$

$$N_{kt} = \left(\int_0^1 N_{kt}(i)^{\frac{\varepsilon_w - 1}{\varepsilon_w}} di \right)^{\frac{\varepsilon_w}{\varepsilon_w - 1}}$$

Implying the FOC,

$$N_{kt}(i) = \left(\frac{W_t(i)}{W_t} \right)^{-\varepsilon_w} N_{kt}$$

where,

$$W_t = \left(\int_0^1 W_t(i)^{1-\varepsilon_w} di \right)^{\frac{1}{1-\varepsilon_w}}$$

Moreover, the firm chooses its price solving this problem,

$$\max \quad P_{kt}Y_{kt} - W_t\left(\frac{Y_{kt}}{A_t}\right)^{\frac{1}{1-\alpha}}$$

$$s.t.$$

$$Y_{kt} = \left(\frac{P_{kt}}{P_t}\right)^{-\varepsilon} Y_t$$

the resulting FOC in a symmetric equilibrium is,

$$P_t = \frac{\varepsilon}{\varepsilon - 1} MC_t$$

where,

$$MC_t = \frac{1}{1-\alpha}\frac{W_t}{A_t^{\frac{1}{1-\alpha}}} Y_t^{\frac{\alpha}{1-\alpha}}$$

1.3 Labor Unions

There is a continuum (in the unit interval) of labor unions each one specialized in a given type of labor $i \in [0, 1]$. They face the downward sloping demand from firms,

$$N_t(i) = \left(\frac{W_t(i)}{W_t}\right)^{-\varepsilon_w} N_t$$

They maximize the discounted net flow of utility from working. They are subject to a Calvo (1983) type constraint to adjust nominal wages. We assume wages grow at rate g_y when they are not adjusted (this is just to simplify detrending and not strictly necessary). The problem of the union is shown below,

$$\max_{W_t^*(i)} \mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta)^k Z_{t+k}\left[\frac{W_t^*(i)(1+g_y)^k N_{t+k}(i) MU_{t+k}}{P_{t+k}} - \chi \frac{N_{t+k}^{1+\varphi}(i)}{1+\varphi}\right]$$

$$s.t.$$

$$N_{t+k}(i) = \left(\frac{W_t^*(i)(1+g_y)^k}{W_{t+k}}\right)^{-\varepsilon_w} N_{t+k}$$

And it implies a FOC,

$$\mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta)^k Z_{t+k} \left[(1-\varepsilon_w) \frac{W_t^*(i)(1+g_y)^k}{P_{t+k}} + \varepsilon_w \frac{\chi N_{t+k}(i)^{\varphi}}{MU_{t+k}} \right] N_{t+k}(i) MU_{t+k} = 0$$

Given that all unions face the same problem, all of them set the same nominal wage when they are able to: $W_t^* = W_t^*(i)$. The average nominal wage is given by,

$$W_t = \left(\int_0^1 W_t(i)^{1-\varepsilon_w} di \right)^{\frac{1}{1-\varepsilon_w}} = \left(\theta \left(W_{t-1}(1+g_y) \right)^{1-\varepsilon_w} + (1-\theta)(W_t^*)^{1-\varepsilon_w} \right)^{\frac{1}{1-\varepsilon_w}}$$

1.4 Market Clearing and Monetary Authority

Up to a first order approximation, goods market clearing is represented by

$$Y_t = A_t N_t^{1-\alpha} = C_t$$

Also, bonds market clearing is,

$$B_t = 0$$

Finally, to close the model we assume a Taylor rule,

$$R_t = R_{ss} \Pi_t^{\phi_\pi} \exp(\sigma_i \nu_t)$$

where $\sigma_i \nu_t$ is a normal iid shocks, and $R_s s$ is the steady state gross nominal interest rate.

2 Model with Government Spending Details

Let's consider the case adding government spending shocks. In particular, assume that government consumption follows the rule,

$$\log\left(\frac{G_t}{(1+g_y)^t}\right) = (1-\rho_g)\log(\bar{G}) + \rho_g \log\left(\frac{G_{t-1}}{(1+g_y)^{t-1}}\right) + \rho_{gy}\log\left(\frac{Y_{t-1}/\bar{Y}}{(1+g_y)^{t-1}}\right) + \sigma_g \varepsilon_{gt}$$

in a log-linearized version,

$$g_t = \rho_g g_{t-1} + \rho_{gy} y_{t-1} + \sigma_g \varepsilon_{gt}$$

Also, now the resource constraint is,

$$y_t = \frac{C}{Y}c_t + \frac{G}{Y}g_t$$

Introducing these two modifications to the baseline case we get the following model,

$$\begin{aligned}
-i_t &= -mu_t + \mathbb{E}_t mu_{t+1} - \mathbb{E}_t \pi_{t+1} + \sigma_z \left(\mathbb{E}_t z_{t+1} - z_t\right) \\
mu_t &= -\frac{1+g_y}{1+g_y-h}\left(\frac{Y}{C}y_t - \frac{G}{C}g_t\right) + \frac{h}{1+g_y-h}\left(\frac{Y}{C}y_{t-1} - \frac{G}{C}g_{t-1}\right) \\
\pi_t^w &= -\kappa^w \mu_t^w + \beta \mathbb{E}_t \pi_{t+1}^w \\
i_t &= \phi_\pi \pi_t + \sigma_i \nu_t \\
\mu_t^w &= \frac{1+\varphi}{1-\alpha}a_t - \frac{\alpha+\varphi}{1-\alpha}y_t + mu_t \\
\frac{1}{1-\alpha}(a_t - a_{t-1}) - \frac{\alpha}{1-\alpha}\left(y_t - y_{t-1}\right) &= \pi_t^w - \pi_t \\
a_t &= a_{t-1} + \sigma_a \varepsilon_{at} \\
g_t &= \rho_g g_{t-1} + \rho_{gy} y_{t-1} + \sigma_g \varepsilon_{gt}
\end{aligned}$$

Taking first differences we get the system of equations used to infer the method to obtain potential GDP,

$$-\phi_\pi \pi_t - \sigma_i \nu_t = \mathbb{E}_t \Delta mu_{t+1} - \mathbb{E}_t \pi_{t+1} + \sigma_z \left(\mathbb{E}_t z_{t+1} - z_t\right) \tag{2.1}$$

$$\Delta mu_t = -\frac{1+g_y}{1+g_y-h}\left(\frac{Y}{C}\Delta y_t - \frac{G}{C}\Delta g_t\right) + \frac{h}{1+g_y-h}\left(\frac{Y}{C}\Delta y_{t-1} - \frac{G}{C}\Delta g_{t-1}\right) \tag{2.2}$$

$$\pi_t^w = -\kappa^w \mu_t^w + \beta \mathbb{E}_t \pi_{t+1}^w \tag{2.3}$$

$$\begin{aligned} \mu_t^w - \mu_{t-1}^w = &-\left(\frac{1+g_y}{1+g-h}\frac{Y}{C} + \frac{\alpha+\varphi}{1-\alpha}\right)\Delta y_t + \frac{h}{1+g_y-h}\frac{Y}{C}\Delta y_{t-1} + \frac{1+\varphi}{1-\alpha}\sigma_a \varepsilon_{at} \\ &\frac{1+g_y}{1+g_y-h}\frac{G}{C}\Delta g_t - \frac{h}{1+g_y-h}\frac{G}{C}\Delta g_{t-1} \end{aligned} \tag{2.4}$$

$$-\frac{\alpha}{1-\alpha}\Delta y_t = \pi_t^w - \pi_t - \frac{1}{1-\alpha}\sigma_a \varepsilon_{at} \tag{2.5}$$

$$\Delta g_t = \rho_g \Delta g_{t-1} + \rho_{gy} \Delta y_{t-1} + \sigma_g \left(\varepsilon_{gt} - \varepsilon_{gt-1}\right) \tag{2.6}$$

3 More Extensions

3.1 Government Spending

In this section, we test how the method and results change when we incorporate government spending as a factor affecting potential GDP. To do so we include in our baseline model a fiscal authority that collects lump-sum taxes, follows a balanced budget, and decides the amount of log government spending g_t (as log-deviation from the detrended steady state) according to the following fiscal rule,

$$g_t = \rho_g g_{t-1} + \rho_{gy} y_{t-1} + \sigma_g \varepsilon_{gt} \tag{3.7}$$

where y_{t-1} represents detrended lagged GDP and ε_t^g is a standard normally distributed government spending shock. This is a common way of modeling fiscal policy in DSGE models which follows Blanchard and Perotti (2002) and assumes that g_t is only affected by lags of macroeconomic variables. This assumption is justified by the usual legislative and implementation lags related to fiscal policy.

The rest of the baseline model remains unchanged. It can be shown that potential GDP is now given by,

$$\Delta y_t^p = \theta_3^g \Delta g_{t-1}^p + \theta_2^g \sigma_g \Delta\varepsilon_{gt} + \theta_1^g \Delta y_{t-1}^p + \theta_0^g \varepsilon_{at} \tag{3.8}$$

with,

$$\theta_0^g \equiv \frac{\frac{1+\varphi}{1-\alpha}\sigma_a}{\frac{1+g_y}{1+g_y-h} + \frac{\alpha+\varphi}{1-\alpha}} \qquad \theta_1^g \equiv \frac{\frac{h}{1+g_y-h}\frac{Y}{C} + \rho_{gy}\frac{1+g_y}{1+g_y-h}\frac{G}{C}}{\frac{1+g_y}{1+g_y-h}\frac{Y}{C} + \frac{\alpha+\varphi}{1-\alpha}}$$

$$\theta_2^g \equiv \frac{\frac{1+g_y}{1+g_y-h}\frac{G}{C}}{\frac{1+g_y}{1+g_y-h}\frac{Y}{C} + \frac{\alpha+\varphi}{1-\alpha}} \qquad \theta_3^g \equiv \frac{\frac{-h}{1+g_y-h}\frac{G}{C} + \rho_g\frac{1+g_y}{1+g_y-h}\frac{G}{C}}{\frac{1+g_y}{1+g_y-h}\frac{Y}{C} + \frac{\alpha+\varphi}{1-\alpha}}$$

where $\frac{Y}{C}$ and $\frac{G}{C}$ are the steady-state output-consumption and government spending-consumption ratios. Moreover, the growth of potential government spending is given by,

$$\Delta g_t^p = \rho_g \Delta g_{t-1}^p + \rho_{gy}\Delta y_{t-1}^p + \sigma_g \Delta\varepsilon_{gt} \tag{3.9}$$

It is clear that potential output now depends on productivity and fiscal policy shocks. The reason why government spending affects potential output has to do with labor supply wealth effects. Specifically, an increase in government spending means higher taxes and lower consumption under flexible prices. This in turn implies lower demand for leisure and a shift to the right of labor supply increasing potential hours and GDP.

Notice that fiscal rule (3.7) is a model equation that depends only on observables: government spending and GDP. Hence, we can estimate the rule's parameters ρ_g and ρ_{gy}, and the fiscal policy shocks $\sigma_g \varepsilon_t^g$ through a simple OLS estimation of the equation. This is actually a first necessary step to get potential GDP estimates using our method. After estimating the fiscal rule, it is straightforward to infer potential GDP using the result in the following proposition,

Proposition 1. *Using a model with wage rigidities and government spending,* θ_0^g*,* θ_1^g*,* θ_2^g*,* θ_3^g *and*

ε_{at} in equation (3.8) can be estimated from the following system,

$$\begin{bmatrix} \Delta y_t \\ \mu_t^w \end{bmatrix} = \mathbf{B} \begin{bmatrix} \Delta y_{t-1} \\ \mu_{t-1}^w \\ \Delta g_{t-1} \\ \sigma_g \varepsilon_{gt} \\ \sigma_g \varepsilon_{gt-1} \end{bmatrix} + \mathbf{C} \begin{bmatrix} \varepsilon_{at} \\ \xi_t \end{bmatrix} \tag{3.10}$$

where ξ_t is a weighted average of demand shocks. In particular, letting c_{ij} and b_{ij} be the elements of matrices $\mathbf{B}$ and $\mathbf{C}$,

$$\theta_0^g = c_{11} - \frac{c_{21}c_{12}}{c_{22}} \qquad \theta_1^g = b_{11} - \frac{b_{21}c_{12}}{c_{22}}$$
$$\theta_2^g = b_{13} - \frac{b_{23}c_{12}}{c_{22}} \qquad \theta_3^g = b_{14} - \frac{b_{24}c_{12}}{c_{22}}$$

And ε_{at} can be calculated using forecast errors and $\mathbf{C}$.

Proof. See Appendix 4. □

As it is clear from the previous proposition, the baseline method is modified by simply adding three data series in the system: lagged government spending growth rate, and current and lagged government spending shocks. Note that, as it is typical in DSGE models, we assume that fiscal shocks are not subject to measurement error. As a consequence, if measurement error is important, our estimates of potential GDP can be significantly biased. Moreover, Blanchard and Perotti (2002) identification strategy can introduce an additional bias if fiscal shocks ε_{gt} are anticipated or not totally unexpected.[1] Due to these concerns, we present the results in this section as a robustness check and the reader should take into account the mentioned shortcomings.

We present the results in figure 1. The earlier results are broadly confirmed after incorporating government spending in our baseline model. There is a high correlation between our estimated output gap and the one estimated by the CBO. Further, we observe an important difference between CBO's and our series during and after the Great Recession. As before, our results imply an increase

[1] See Ramey (2011).

in potential GDP during and right after the Great Recession but a poor potential growth afterwards. Conversely, CBO's estimates suggest a slowdown in the growth rate during and after the recession with a subsequent acceleration.

A new result that emerges in this case is the large difference between this new series and the one estimated using the baseline method from 1950 until around 1975. This difference has to do with large government spending shocks associated with the Korean and Vietnam wars. According to our estimates, these large fiscal shocks explain part of the increase in GDP that was previously attributed to TFP shocks in our baseline method. Hence, part of the increase on GDP that was previously associated with positive TFP shocks is now explained by government spending shocks. Given that TFP shocks are the main factors affecting potential GDP, this change in the relative importance of the shocks has an impact on estimated potential GDP. Particularly, our method including fiscal shocks predicts a lower TFP and potential GDP growth during this period, which are associated with higher output gaps.

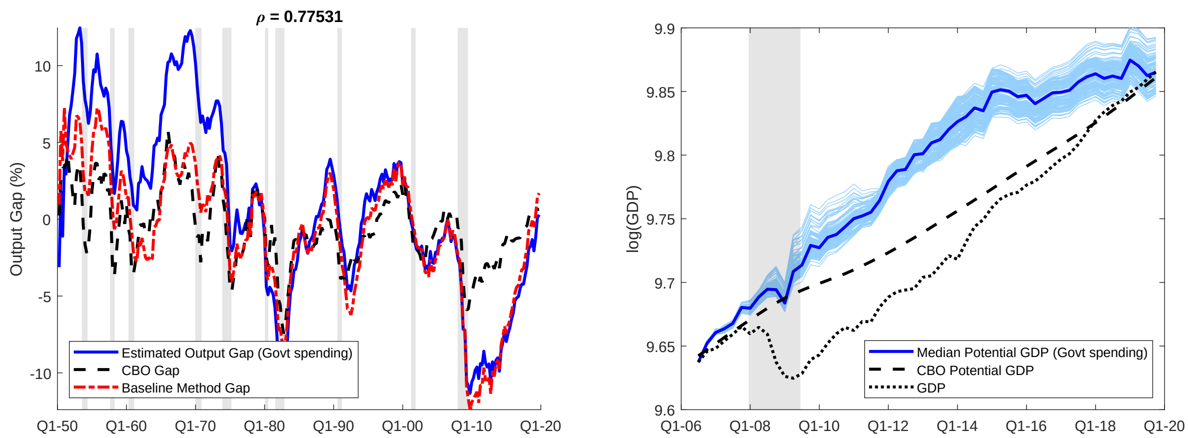
3.2 A COVID-19 labor supply shock

The COVID-19 pandemic and related lockdown was a combination of important demand and supply shocks. One the one hand, demand decreased because of the increasing uncertainty about the future evolution of the pandemic and the lockdown measures that forbade consumption of certain services. On the other, supply decreased because lockdown measures reduced labor supply in the economy.[2]

The baseline method and extensions we have developed so far are able to separately identify productivity shocks and a linear combination of "demand shocks" (ξ_t), but they implicitly assume no labor supply shocks. The reason behind this assumption is that exogenous labor supply changes are typically related to demographic or institutional issues that are relevant at low frequency only. Therefore, we considered that labor supply shocks were included in the low-frequency component of the wage markup series. As a result, we assumed we were dropping labor supply shocks from the analysis after detrending the wage markup series. Including the COVID-19 pandemic and its negative short-run labor supply shock in our analysis forces us to modify our assumptions, at least for the observations after 2020Q1 in the US.

[2] Brinca et al. (2020) find that the impact of COVID in the labor market can be mostly explained by labor supply shocks.

Figure 1: Including Government Spending: Output Gap and Potential GDP



(a) Output Gap vs CBO

(b) Potential GDP during Great Recession

Note: Panel (a) shows the output gap computed using the method assuming wage rigidities and including government spending in the underlying model. We also include CBO's output cap for comparison. Panel (b) shows potential GDP series computed with the same method. Light blue lines in panel (b) highlight different estimates using different samples starting at different dates, using as starting dates quarters from 1950Q1 to 1990Q1. The thick dark blue line represents the median value across all estimates. See data appendix **??** *for details.*

We modify the baseline model and incorporate a COVID-19 labor supply shock χ_t to analyze the COVID-19 recession. In particular, we assume that the disutility of labor is exogenously affected by χ_t and that the changes in this parameter are stochastic and iid, $\Delta\chi_t = \sigma_\chi \varepsilon_{\chi t}$. Keeping the rest of the model unchanged, it is easy to show that potential GDP is now given by,[3]

$$\Delta y_t^p = \theta_1 \Delta y_{t-1}^p + \theta_0 \varepsilon_{at} + \theta_2 \Delta\chi_t \tag{3.11}$$

where θ_0 and θ_1 are defined as in the paper, and $\theta_2 \equiv -\left(\frac{1+g_y}{1+g_y-h} + \frac{\alpha+\varphi}{1-\alpha}\right)^{-1}$. Hence, we now have to estimate an additional parameter, θ_2, and a new shock series $\Delta\chi_t$. The next proposition describes how to estimate the θ's and productivity shocks ε_{at} with this model.

Proposition 2. *θ_0, θ_1, θ_2 and ε_{at} in equation (3.11) can be estimated from the following Structural*

[3] See appendix 5 for details.

Vector Autoregression (SVAR) estimation,

$$\begin{bmatrix} \Delta y_t \\ \mu_t^w \end{bmatrix} = \mathbf{B} \begin{bmatrix} \Delta y_{t-1} \\ \mu_{t-1}^w \end{bmatrix} + \mathbf{C} \begin{bmatrix} \varepsilon_{at} \\ \xi_t \end{bmatrix} \tag{3.12}$$

where Δy_t is the GDP growth rate and ξ_t is a weighted average of demand shocks and the labor supply shock. In particular, letting c_{ij} and b_{ij} for $i, j = \{1, 2\}$ be the elements of the 2×2 matrices $\mathbf{B}$ *and* $\mathbf{C}$*,*

$$\theta_0 = c_{11} - \frac{c_{21}c_{12}}{c_{22}} \qquad \theta_1 = b_{11} - \frac{b_{21}c_{12}}{c_{22}}$$
$$\theta_2 = \frac{c_{12}}{c_{22}}$$

And ε_{at} can be calculated using forecast errors and $\mathbf{C}$*.*

Proof. See Appendix 5. □

Proposition 2 tells us that the SVAR estimation with labor supply shocks is the same as the one of the baseline method. We need to estimate the baseline SVAR, and after doing so we can recover the θ's and ε_{at} in equation (3.11). After running this estimation the only remaining step to estimate Δy_t^p is to infer COVID-19 labor-supply shocks ($\Delta \chi_t$) from the data. This last step is obviously challenging. However, using our structural model and introducing reasonable assumptions we can actually put discipline in our estimates.

Using the labor supply equation from the household and the definition of μ_t^w in (**??**) we get the following expression,[4]

$$\mu_t^w = \varphi(l_t - n_t) + \chi_t$$

where l_t is total hours supplied by the household, n_t is total hours worked and χ_t is the labor disutility shock. The last expression suggests a clear strategy to estimate labor supply shocks. Assuming a value for the inverse Frisch elasticity we can use data on wage markups, labor supply

[4]See Appendix 5 for details.

and hours worked to obtain χ_t as a residual. This is a simple method, but it also has clear shortcomings. First, there is no data available on hours supplied and, therefore, we need to use as proxy for $l_t - n_t$ the log-difference between the labor participation rate and the employment ratio. This is a reasonable way to proceed but it might add significant measurement error. Second, labor disutility shocks and wage markup shocks enter in exactly the same way in the household labor supply equation. Hence, if there are exogenous changes in the market power of labor unions, our estimate of χ_t may be contaminated by these other shocks.

Figure 2 shows the data series for $\Delta\mu_t^w - \varphi(\Delta l_t - \Delta n_t)$ assuming $\varphi = 1$, the same value we assumed to compute μ_t^w.[5] From the figure we can see a large increase in the series after 2020Q1 followed by an important reduction, which is correlated with the lockdown severity. Notice that before the pandemic there are changes in the series, but these are an order of magnitude smaller. Taking into account the mentioned shortcomings regarding the estimation of $\Delta\chi_t$, we take a conservative approach and assume that fluctuations in $\Delta\mu_t^w - \varphi(\Delta l_t - \Delta n_t)$ before the pandemic are explained by either measurement error or short-run wage markup shocks and, therefore, do not represent exogenous changes in labor disutility. In addition, we assume that the large changes in $\Delta\mu_t^w - \varphi(\Delta l_t - \Delta n_t)$ after 2020Q1 are entirely explained by fluctuations in $\Delta\chi_t$. Hence, our labor disutility shock is $\Delta\chi_t = 0$ before the pandemic and $\Delta\chi_t = \Delta\mu_t^w - \varphi(\Delta l_t - \Delta n_t)$ since 2020Q1.

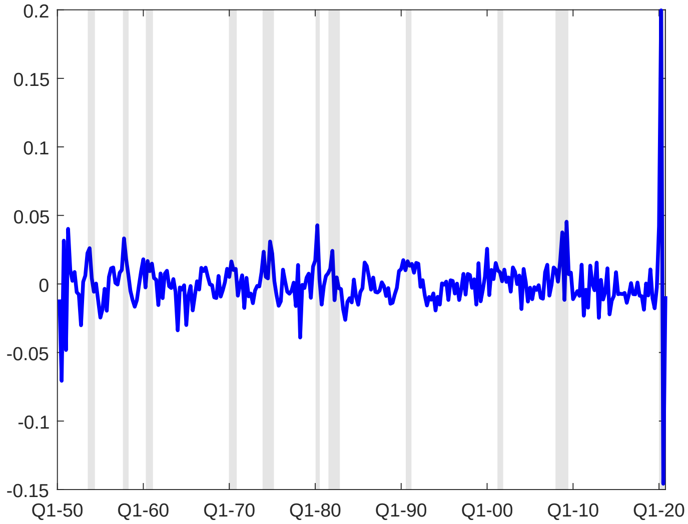
Figure 3 shows potential GDP estimates extending the analysis up to 2020Q4 and incorporating the COVID-19 shock to potential GDP.[6] The results show a significant drop in potential GDP as a consequence of the pandemic. However, the decrease in potential GDP is less pronounced than the decrease in GDP and, as a cosequence, the output gap became negative in 2020. As shown in the figure, the results are robust to picking different samples with different starting points.

Labor supply shocks play an important role in the sudden decrease and posterior increase in potential GDP. This implies that computation of potential output during the pandemic requires incorporating this kind of shocks. This simple extension shows that it is indeed simple to add high-frequency changes in labor supply related to COVID-19.

[5] Results are qualitatively similar if we assume other values for φ.

[6] Given the magnitude of the COVID shocks, we excluded the year 2020 from the SVAR estimation.

Figure 2: Data series for $\Delta\mu_t^w - \varphi(\Delta l_t - \Delta n_t)$

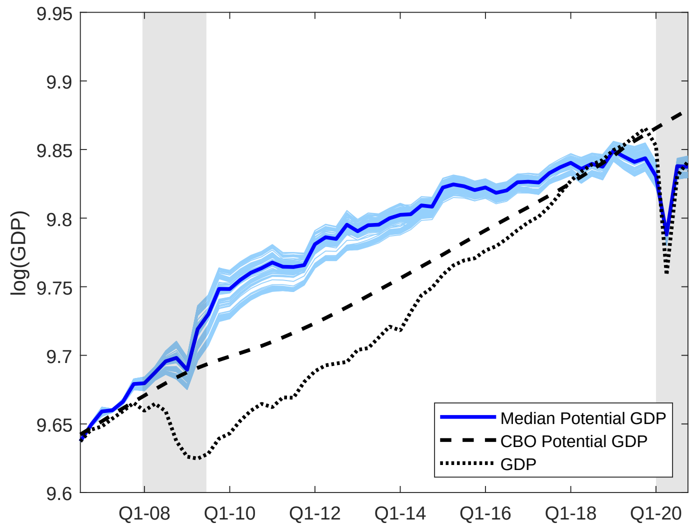


4 Proof of proposition 1

The log-linearized version of the model in first-differences is given by the following system of equations.

$$
\begin{aligned}
-\phi_\pi \pi_t - \sigma_i \nu_t &= \mathbb{E}_t \Delta mu_{t+1} - \mathbb{E}_t \pi_{t+1} + \sigma_z \left(\mathbb{E}_t z_{t+1} - z_t\right) && (4.1)\\
\Delta mu_t &= -\frac{1+g_y}{1+g_y-h}\left(\frac{Y}{C}\Delta y_t - \frac{G}{C}\Delta g_t\right) + \frac{h}{1+g_y-h}\left(\frac{Y}{C}\Delta y_{t-1} - \frac{G}{C}\Delta g_{t-1}\right) && (4.2)\\
\pi_t^w &= -\kappa^w \mu_t^w + \beta \mathbb{E}_t \pi_{t+1}^w && (4.3)\\
\mu_t^w - \mu_{t-1}^w &= -\left(\frac{1+g_y}{1+g-h}\frac{Y}{C} + \frac{\alpha+\varphi}{1-\alpha}\right)\Delta y_t + \frac{h}{1+g_y-h}\frac{Y}{C}\Delta y_{t-1} + \frac{1+\varphi}{1-\alpha}\sigma_a \varepsilon_{at}\\
&\quad \frac{1+g_y}{1+g_y-h}\frac{G}{C}\Delta g_t - \frac{h}{1+g_y-h}\frac{G}{C}\Delta g_{t-1} && (4.4)\\
-\frac{\alpha}{1-\alpha}\Delta y_t &= \pi_t^w - \pi_t - \frac{1}{1-\alpha}\sigma_a \varepsilon_{at} && (4.5)\\
\Delta g_t &= \rho_g \Delta g_{t-1} + \rho_{gy}\Delta y_{t-1} + \sigma_g \left(\varepsilon_{gt} - \varepsilon_{gt-1}\right) && (4.6)
\end{aligned}
$$

Figure 3: Potential GDP during COVID-19



Note: Light blue lines highlight different estimates using different samples starting at different dates, using as starting dates quarters from 1950Q1 to 1990Q1. The thick dark blue line represents the median value across all estimates. See data appendix **??** *for details.*

The first equation in the system to be estimated comes from combining (4.4) and (4.6),

$$\mu_t^w - \mu_{t-1}^w = -\left(\frac{1+g_y}{1+g-h}\frac{Y}{C} + \frac{\alpha+\varphi}{1-\alpha}\right)\Delta y_t + \left[\frac{h}{1+g_y-h}\frac{Y}{C} + \frac{1+g_y}{1+g_y-h}\frac{G}{C}\rho_{gy}\right]\Delta y_{t-1} + \frac{1+\varphi}{1-\alpha}\sigma_a\varepsilon_{at}$$
$$+ \frac{1+g_y}{1+g_y-h}\frac{G}{C}\sigma_g(\varepsilon_{gt}-\varepsilon_{gt-1}) + \frac{G}{C}\left[\frac{-h}{1+g_y-h} + \frac{1+g_y}{1+g_y-h}\rho_g\right]\Delta g_{t-1} \quad (4.7)$$

The second equation comes from the fact that the wage mark-up is a function of the state variables and shocks in the model. In this version of the model the state variables are μ_{t-1}^w, Δy_{t-1}, Δg_{t-1}, and ε_{gt-1}. Moreover, the shocks are ε_{at}, z_t, ν_t, and ε_{gt}. Hence, we can write the following expression,

$$\mu_t^w = \gamma_a\varepsilon_{at} + \gamma_\xi\xi_t + \gamma_\mu\mu_{t-1}^w + \gamma_y\Delta y_{t-1} + \gamma_g\Delta g_{t-1} + \gamma_{\varepsilon_g}^0\sigma_g\varepsilon_{gt} + \gamma_{\varepsilon_g}^1\sigma_g\varepsilon_{gt-1} \quad (4.8)$$

where ξ_t is just a linear combination of preference shocks z_t and monetary policy shocks ν_t. Equations (4.7) and (4.8) form the following system,

$$\begin{bmatrix} \Delta y_t \\ \mu_t^w \end{bmatrix} = \mathbf{B} \begin{bmatrix} \Delta y_{t-1} \\ \mu_{t-1}^w \\ \Delta g_{t-1} \\ \sigma_g \varepsilon_{gt} \\ \sigma_g \varepsilon_{gt-1} \end{bmatrix} + \mathbf{C} \begin{bmatrix} \varepsilon_{at} \\ \xi_t \end{bmatrix}$$

where,

$$\mathbf{B} \equiv \begin{bmatrix} \frac{\frac{h}{1+g_y-h}\frac{Y}{C} - \gamma_y + \rho_{gy}\frac{1+g_y}{1+g_y-h}\frac{G}{C}}{\frac{1+g_y}{1+g_y-h}\frac{Y}{C} + \frac{\alpha+\varphi}{1-\alpha}} & \frac{1-\gamma_\mu}{\frac{1+g_y}{1+g_y-h}\frac{Y}{C} + \frac{\alpha+\varphi}{1-\alpha}} & \frac{\frac{-h}{1+g_y-h}\frac{G}{C} - \gamma_g + \rho_g\frac{1+g_y}{1+g_y-h}\frac{G}{C}}{\frac{1+g_y}{1+g_y-h}\frac{Y}{C} + \frac{\alpha+\varphi}{1-\alpha}} & \frac{\frac{1+g_y}{1+g_y-h}\frac{G}{C} - \gamma_{\varepsilon_g}^0}{\frac{1+g_y}{1+g_y-h}\frac{Y}{C} + \frac{\alpha+\varphi}{1-\alpha}} & -\frac{\frac{1+g_y}{1+g_y-h}\frac{G}{C} + \gamma_{\varepsilon_g}^1}{\frac{1+g_y}{1+g_y-h}\frac{Y}{C} + \frac{\alpha+\varphi}{1-\alpha}} \\ \gamma_y & \gamma_\mu & \gamma_g & \gamma_{\varepsilon_g}^0 & \gamma_{\varepsilon_g}^1 \end{bmatrix}$$

$$\mathbf{C} \equiv \begin{bmatrix} \frac{\sigma_a \frac{1+\varphi}{1-\alpha} - \gamma_a}{\frac{1+g_y}{1+g_y-h}\frac{Y}{C} + \frac{\alpha+\varphi}{1-\alpha}} & -\frac{\gamma_\xi}{\frac{1+g_y}{1+g_y-h}\frac{Y}{C} + \frac{\alpha+\varphi}{1-\alpha}} \\ \gamma_a & \gamma_\xi \end{bmatrix}$$

Using the previous matrices it is straightforward to check that,

$$\theta_0^g \equiv \frac{\frac{1+\varphi}{1-\alpha}\sigma_a}{\frac{1+g_y}{1+g_y-h} + \frac{\alpha+\varphi}{1-\alpha}} = c_{11} - \frac{c_{21}c_{12}}{c_{22}}$$

$$\theta_1^g \equiv \frac{\frac{h}{1+g_y-h}\frac{Y}{C} + \rho_{gy}\frac{1+g_y}{1+g_y-h}\frac{G}{C}}{\frac{1+g_y}{1+g_y-h}\frac{Y}{C} + \frac{\alpha+\varphi}{1-\alpha}} = b_{11} - \frac{b_{21}c_{12}}{c_{22}}$$

$$\theta_2^g \equiv \frac{\frac{1+g_y}{1+g_y-h}\frac{G}{C}}{\frac{1+g_y}{1+g_y-h}\frac{Y}{C} + \frac{\alpha+\varphi}{1-\alpha}} = b_{14} - \frac{b_{24}c_{12}}{c_{22}}$$

$$\theta_3^g \equiv \frac{\frac{-h}{1+g_y-h}\frac{G}{C} + \rho_g\frac{1+g_y}{1+g_y-h}\frac{G}{C}}{\frac{1+g_y}{1+g_y-h}\frac{Y}{C} + \frac{\alpha+\varphi}{1-\alpha}} = b_{13} - \frac{b_{23}c_{12}}{c_{22}}$$

5 Proof of proposition 2

After detrending and loglinearizing, the baseline model with labor supply shocks can be summarized by the following system,

$$-\phi_\pi \pi_t - \sigma_i \nu_t = \mathbb{E}_t \Delta mu_{t+1} - \mathbb{E}_t \pi_{t+1} + \sigma_z \left(\mathbb{E}_t z_{t+1} - z_t\right) \tag{5.1}$$

$$\Delta mu_t = -\frac{1+g_y}{1+g_y-h}\Delta y_t + \frac{h}{1+g_y-h}\Delta y_{t-1} \tag{5.2}$$

$$\pi_t^w = -\kappa^w \mu_t^w + \beta \mathbb{E}_t \pi_{t+1}^w \tag{5.3}$$

$$\mu_t^w - \mu_{t-1}^w = -\left(\frac{1+g_y}{1+g-h} + \frac{\alpha+\varphi}{1-\alpha}\right)\Delta y_t + \frac{h}{1+g_y-h}\Delta y_{t-1} + \frac{1+\varphi}{1-\alpha}\sigma_a \varepsilon_{at} \tag{5.4}$$

$$\frac{1}{1-\alpha}\sigma_a \varepsilon_{at} - \frac{\alpha}{1-\alpha}\Delta y_t = \pi_t^w - \pi_t \tag{5.5}$$

$$(1-\alpha)\Delta n_t = \Delta y_t - \sigma_a \varepsilon_{at} \tag{5.6}$$

$$\mu_t^w - \mu_{t-1}^w = \varphi(\Delta l_t - \Delta n_t) + \Delta \chi_t \tag{5.7}$$

where the last expression (5.7) comes from taking first differences and combining labor demand from firms,

$$w_t - p_t = \frac{1}{1-\alpha}a_t - \frac{\alpha}{1-\alpha}y_t$$

and labor supply,

$$0 = w_t - p_t - \varphi l_t + mu_t - \chi_t$$

Potential GDP growth in this model is the growth rate when $\Delta\mu_t^w$ is only explained by exogenous shifts in labor supply due to changes in disutility of labor, that is, $\Delta\mu_t^w = \Delta\chi_t$. Incorporating the fact that $\Delta\mu_t^w = \Delta\chi_t$ into (5.4) and rearranging we get,

$$\Delta y_t^p = \frac{\Omega_2}{\Omega_1}\Delta y_{t-1}^p + \frac{\Omega_3}{\Omega_1}\varepsilon_{at} - \frac{1}{\Omega_1}\Delta\chi_t$$

where,

$$\Omega_1 \equiv \frac{1+g_y}{1+g_y-h} + \frac{\alpha+\varphi}{1-\alpha}$$
$$\Omega_2 \equiv \frac{h}{1+g_y-h}$$
$$\Omega_3 \equiv \frac{1+\varphi}{1-\alpha}\sigma_a$$

Like in the previous cases, the SVAR is composed by equation (5.4) and an equation describing μ_t^w as a function of exogenous shocks and state variables:

$$\mu_t^w = \gamma_a \varepsilon_{at} + \gamma_z z_t + \gamma_\nu \nu_t + \gamma_\chi \Delta\chi_t + \gamma_\mu \mu_{t-1}^w + \gamma_y \Delta y_{t-1}$$

Defining now $\xi_t = \frac{\gamma_z}{\sqrt{\gamma_z^2+\gamma_\nu^2+\gamma_\chi^2}} z_t + \frac{\gamma_\nu}{\sqrt{\gamma_z^2+\gamma_\nu^2+\gamma_\chi^2}} \nu_t + \frac{\gamma_\chi}{\sqrt{\gamma_z^2+\gamma_\nu^2+\gamma_\chi^2}} \Delta\chi_t$ and $\gamma_\xi = \sqrt{\gamma_z^2+\gamma_\nu^2+\gamma_\chi^2}$ then,

$$\mu_t^w = \gamma_a \varepsilon_{at} + \gamma_\xi \xi_t + \gamma_\mu \mu_{t-1}^w + \gamma_y \Delta y_{t-1} \tag{5.8}$$

Now combining (5.4), (5.8) and after some algebra we find the same SVAR as the one in Proposition 1 in the paper,

$$\begin{bmatrix} \Delta y_t \\ \mu_t^w \end{bmatrix} = \mathbf{B} \begin{bmatrix} \Delta y_{t-1} \\ \mu_{t-1}^w \end{bmatrix} + \mathbf{C} \begin{bmatrix} \varepsilon_{at} \\ \xi_t \end{bmatrix}$$

where,

$$\mathbf{B} \equiv \begin{bmatrix} \frac{\frac{h}{1+g_y-h} - \gamma_y}{\Omega_1} & \frac{1-\gamma_\mu}{\Omega_1} \\ \gamma_y & \gamma_\mu \end{bmatrix} \qquad \mathbf{C} \equiv \begin{bmatrix} \frac{\Omega_3-\gamma_a}{\Omega_1} & \frac{-\gamma_\xi}{\Omega_1} \\ \gamma_a & \gamma_\xi \end{bmatrix}$$

Using the matrices it is easy to check that,

$$\frac{\Omega_3}{\Omega_1} = c_{11} - \frac{c_{21}c_{12}}{c_{22}} \qquad \frac{\Omega_2}{\Omega_1} = b_{11} - \frac{b_{21}c_{12}}{c_{22}}$$
$$\frac{-1}{\Omega_1} = \frac{c_{12}}{c_{22}}$$

6 Endogenous TFP: Bias and Estimation Method

In this section we propose and test a new method that might be helpful when the bias introduced by TFP endogeneity is important. When TFP is affected by demand shocks ξ_t, the SVAR to be estimated is the following,

$$\begin{bmatrix} \Delta y_t \\ \mu_t^w \end{bmatrix} = \mathbf{B} \begin{bmatrix} \Delta y_{t-1} \\ \mu_{t-1}^w \end{bmatrix} + \mathbf{C} \begin{bmatrix} \eta_a \tilde{\varepsilon}_{at} + \sum_{j=n_1}^{n_2} \eta_j \xi_{t-j} \\ \xi_t \end{bmatrix} \tag{6.1}$$

Given the endogeneity problem, we propose a method using GMM to get consistent estimates. The method uses estimated lagged ξ_t demand shocks as instruments. In particular, given a value for $\mathbf{B}$, we construct the objective function to be minimized in the following steps. Letting $\hat{\mathbf{B}}$ be a guess for the estimate of $\mathbf{B}$,

1. Get forecast errors,

$$\begin{bmatrix} u_t^{\Delta y} \\ u_t^{\mu^w} \end{bmatrix} = \begin{bmatrix} \Delta y_t \\ \mu_t^w \end{bmatrix} - \hat{\mathbf{B}} \begin{bmatrix} \Delta y_{t-1} \\ \mu_{t-1}^w \end{bmatrix} \tag{6.2}$$

2. Use a proxy variable and the forecast errors from the previous step to compute an estimate for $\mathbf{C}$, call it $\hat{\mathbf{C}}$. Use $\hat{\mathbf{C}}$ to estimate structural shocks ξ_t,

$$\begin{bmatrix} \hat{\varepsilon}_{at} \\ \hat{\xi}_t \end{bmatrix} = \hat{\mathbf{C}}^{-1} \begin{bmatrix} u_t^{\Delta y} \\ u_t^{\mu^w} \end{bmatrix}$$

3. Construct moment conditions using the fact that TFP is only affected with a lag of n_1

quarters. Define

$$g_{jt}^{\Delta y} = u_t^{\Delta y} \hat{\xi}_{t-j}$$
$$g_{jt}^{\mu^w} = u_t^{\mu^w} \hat{\xi}_{t-j}$$

and

$$g_t \equiv \left[g_{1t}^{\Delta y}, g_{2t}^{\Delta y}, \ldots, g_{n_1-1,t}^{\Delta y}, g_{1t}^{\mu^w}, g_{2t}^{\mu^w}, \ldots, g_{n_1-1,t}^{\mu^w} \right]'$$

The moment conditions are given as

$$\bar{g} \equiv \frac{1}{N - n_1 + 1} \sum_{t=1+n_1-1}^{N} g_t$$

4. Construct the efficient weighting matrix using the formula below,

$$W = \left(\frac{1}{N - n_1 + 1} \sum_{t=1+n_1-1}^{N} \hat{g}_t \hat{g}_t' - \bar{g}\bar{g}' \right)^{-1}$$

and create the objective function to be minimized in the following way,

$$J = \bar{g}' W \bar{g}$$

Minimization of function J provides a GMM estimator for matrices **B** and **C**. This method performs better than OLS when the source of bias is large.[7] However, we are interested in checking the performance of both OLS and the described GMM methods in a realistic environment. To do so we perform Monte Carlo simulations using as data generating process our full-sample baseline estimation results for matrices **B** and **C**. We run 10,000 simulations of a length of 280 quarters (70

[7]For instance, compared to OLS, GMM is closer to the true parameter values for the case in which TFP is solely determined by lagged demand shocks. Results available upon request.

years) assuming that TFP growth is determined by,

$$\varepsilon_{at} = 0.999987 \;\; \tilde{\varepsilon}_{at} + 0.005 \;\; \xi_{t-5}$$

which means that a negative one-standard-deviation demand shocks ξ_t reduces TFP by 0.5% after 5 quarters. We calibrated this TFP process using the information contained in figures **??**. In the figures, demand shocks affect TFP with a lag of at least 5 quarters, and the maximum change in TFP growth is -0.5%. The simulation results are shown in table 1.

Table 1: Testing Estimation Methods with Endogenous TFP

Parameter	DGP	OLS		GMM	
		Average	[p5, p95]	Average	[p5, p95]
b_{11}	0.368	0.364	[0.269 , 0.455]	0.382	[0.242 , 0.549]
b_{21}	-0.79	-0.79	[-0.953 , -0.624]	-0.786	[-1.073 , -0.509]
b_{12}	-0.004	-0.002	[-0.009 , 0.009]	-0.006	[-0.033 , 0.026]
b_{22}	0.976	0.972	[0.953 , 0.984]	0.968	[0.915 , 1.055]
c_{11}	0.004	0.004	[0.003 , 0.005]	0.006	[0.003 , 0.018]
c_{21}	0.008	0.009	[0.008 , 0.01]	0.014	[0.008 , 0.045]
c_{12}	0.008	0.008	[0.007 , 0.008]	0.008	[0.007 , 0.008]
c_{22}	-0.012	-0.012	[-0.013 , -0.011]	-0.012	[-0.014 , -0.011]

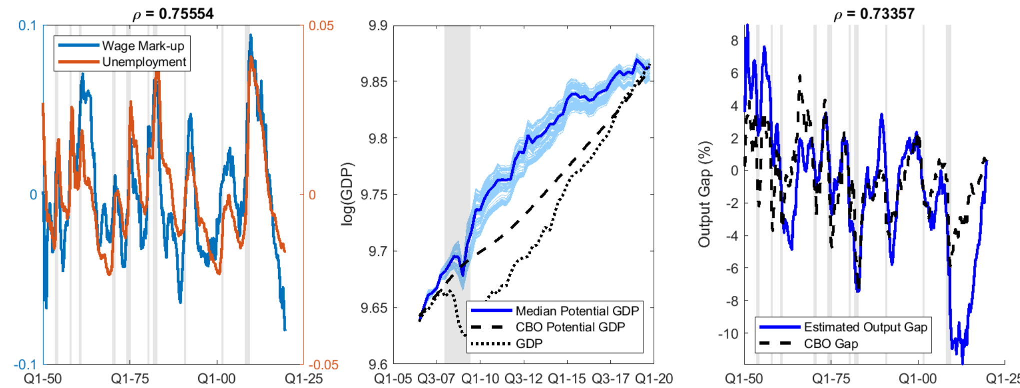
Note: b_{ij} and c_{ij} for $i, j = \{1, 2\}$ are elements of matrix $\mathbf{B}$ and $\mathbf{C}$. The average values and 5 to 95 percentile across the 10,000 simulations are displayed.

Two results emerge from the table. First, OLS estimates perform better than GMM ones in this realistic case, especially **B** estimates. Second, the bias of OLS estimates is small and the true parameter values are always between percentiles 5 and 95 for this estimation method.

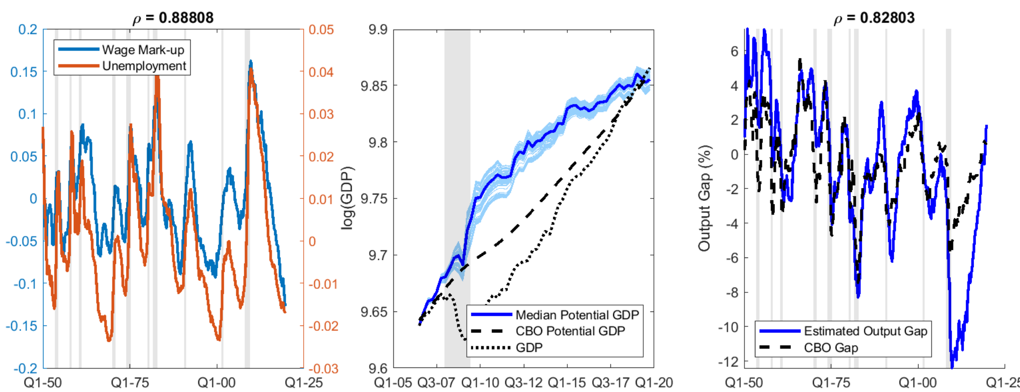
7 Robustness analysis: Frisch elasticity

In this section we perform robustness analysis for the inverse of Frisch elasticity φ. In particular, we present below the wage markup series, potential GDP, and output gap, for three different values of φ: $1/3, 1$ and 2. Recall that $\varphi = 1$ in our main analysis following Galí et al. (2007). As shown in figure 4, our estimates are robust to different values for φ.

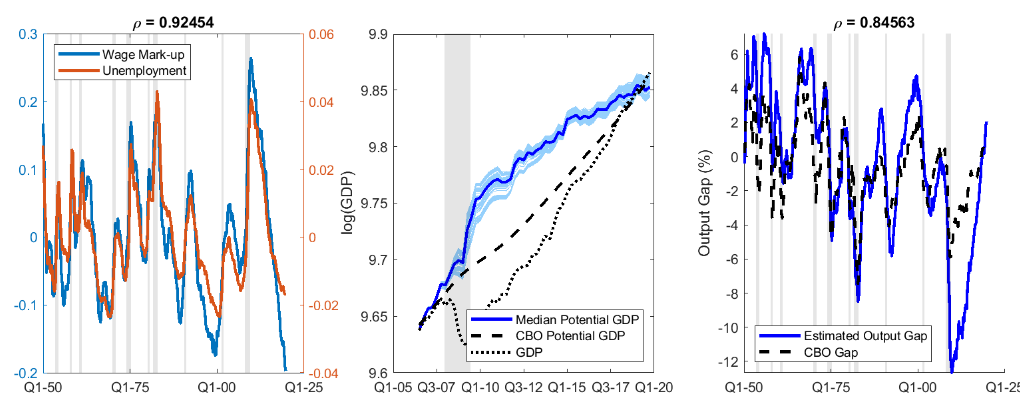
Figure 4: Robustness analysis: Frisch elasticity



(a) $\varphi = 1/3$



(b) $\varphi = 1$



(c) $\varphi = 2$

Note: potential GDP is derived from the baseline method using Fernald's TFP as a proxy.

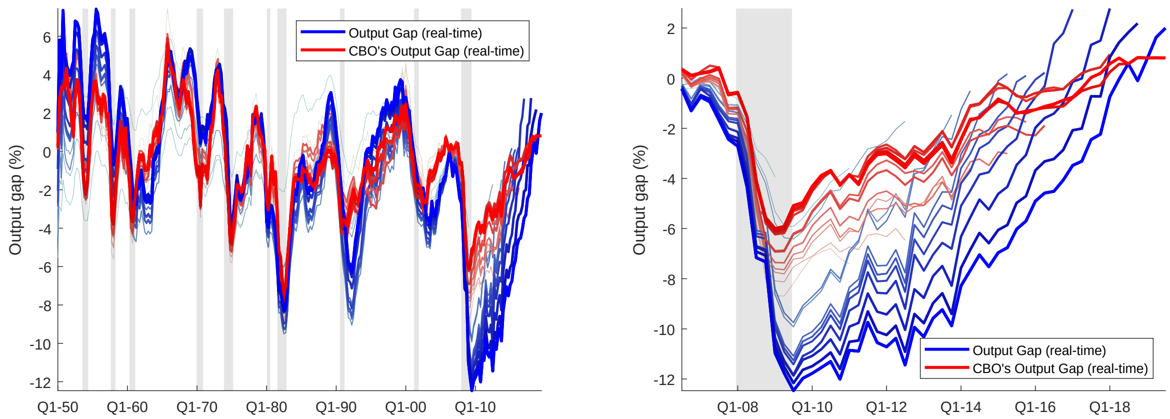
8 Real-time estimation

In this section, we perform real-time analysis for our baseline SVAR estimation using Fernald (2012) TFP series as a proxy. Specifically, we estimate the model as if we had data available at each release date of Fernald (2012) TFP series. For this, we construct data that was available at those 61 release dates – the earliest date is August 16, 2007 and the latest date is February 6, 2020 at which we get the whole sample (1950Q1 - 2019Q4). Using this vintage data set, we estimate potential GDP at each date and compare it with CBO's estimate available at that time.[8]

Figure 5 shows estimated output gap (in blue) and CBO's output gap (in red). The thicker and darker lines represent estimates using most recent data. The two estimates differ starkly during and after the Great Recession as displayed in panel (b). Our estimate points to larger output gaps as data is revised and new data becomes available, whereas CBO suggests smaller output gaps. As discussed, our estimate closely follows the revision and updates in TFP series. In contrast, CBO's estimate is revised and updated to be smaller as the economy recovers.

[8]The source of data is ALFRED [Link here]. In ALFRED, the vintage for Real personal consumption expenditures per capita (A794RX0Q048SBEA) is available back to March 27, 2014. No vintage is available before that date. We find those vintages directly from BEA Data Archive [Link here].

Figure 5: Real-time estimation: Output Gap



(a) Output Gap vs CBO **(b)** Output Gap during and after Great Recession

Note: Panel (a) shows the output gap estimated in real-time using baseline model with Fernald's tfp series (in blue). We also include CBO's output gap for comparison (in red). The Panel (b) highlights different behavior of the two estimates during and after the Great Recession. Estimates with thicker and darker lines are the ones computed using more recent data.

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